# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.,** DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2013**

**ST 1503/ST 1501 – PROBABILITY AND RANDOM VARIABLES**

Date : 14/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

## PART - A

Answer **ALL** the questions: (10 x 2 = 20 Marks)

1. Define Random Experiment.
2. If two events A and B have P(A) = 1/ 3 , P(B) = 3 /4 and P( A □ B) = 11 /12, then find the probability that exactly one of A and B occurs.
3. If two unbiased dice are thrown once, write down the sample space, then find the probability that sum of the points on the upper most faces of the dice fallen is at least 11.
4. The probability that a student passes a Physics test is 2 / 3 and the probability that he passes both a Physics test and an English test is 14 /45. The probability that he passes at least one test is 4 /5. What is the probability that he passes the English test?
5. If A and B are independent events, then show that $\bar{A}$ and $\bar{B}$ are also independent.
6. If for two events A and B , P(A) = 1 /2 , P( B / A) = 1 /3 then find P( A – B ).
7. For two independent events A and B , P(A) = 0.4, P(B) = 0.5, then find P( A □ B).
8. An oil exploration firm finds that 5 % of the test wells it drills yields a deposit of natural gas. If the firm drills 6 wells, what is the probability that at least one well will yield gas ?
9. Define Discrete and Continuous random variables.
10. A box contains 4 white and 6 Red balls. If 2 balls are drawn at random from it, find the mathematical expectation of the number of red balls.

## PART - B

Answer any **FIVE** questions: (5 x 8 = 40 Marks)

11. State and prove addition law of probability, considering 3 non-mutually exclusive events.
12. Three newspapers A, B and C are published in a certain city. It is estimated from a survey that of the adult population: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find what percentage read at least one of the papers?
13. The probability that a contractor will get a plumbing contract is 2/ 3, and the probability that he will not get an electric contract is 5 / 9. If the probability of getting at least one contract is 4 / 5 , what is the probability that he will get both the contracts?
14. A committee of 5 persons is formed from 8 gentlemen and 3 ladies. What is the probability that the committee contains (a) exactly 2 ladies. (b) at least one lady?

15. State and prove Multiplication theorem, considering three events.

16. Two coins are tossed. If $E_1$ is the event " head on first coin ", $E_2$ is the event, ′head on second coin ", and $E_3$ the event " the coins match, both are heads or both tails". Prove that the three events are pair-wise independent events. Are they independent?

17. If t is a non-negative real number, show that the function defined by

$$f(x) = e^{-t}(1 - e^{-t})^{x-1}$$

can represent a probability function of a discrete random variable X assuming the values 1,2,3,… . Find the expectation of X.

18. Show that $C_X''(0)$ = Var [X], where $C_X(t)$ is the cumulant generating function of random variable X and ' denotes differentiation with respect to t.

**PART - C**

Answer any **TWO** questions: (2 x 20 = 40 )

19. For n events $A_1, A_2, \ldots, A_n$, prove that

(i) $P(\bigcap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

(ii) $P(\bigcap_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$ (10 + 10)

20 a) State and prove Baye's Theorem.

b) In a bolt factory, machine $X_1$, $X_2$,and $X_3$ manufacture respectively 20%, 30% and 50% of the total of their output. Of them 5,4 and 2 percent respectively are defective bolts. A bolt is drawn at random from the products and is found to be defective. What is the probability that it was manufactured by machine $X_2$ or $X_3$ ? (10 + 10)

21. a) State and prove Chebychev's inequality.

b) For geometric distribution $p(x) = 2^{-x}$; x = 1,2,3 …. .Prove that Chebychev's inequality gives $P[|X - 2| \leq 2] > 1/2$, while the actual probability is 15 / 16. (10 + 10)

22. a) Let X be a discrete random variable taking values 0,1 and 2 with probabilities p, $1 - 2p$ and p respectively with $0 \leq p \leq 0.5$. Find the value of p for which Var(X) is maximum.

b) For the following probability distribution

$$dF = y_0 \cdot e^{-|x|} dx \quad , -\infty \leq x \leq \infty$$

Find values of $y_0$ and the standard deviation of the distribution. (10 + 10)

**$$$$$$$**